GCE A LEVEL

1305U40-1

Z22-1305U40-1

WEDNESDAY, 8 JUNE 2022 – AFTERNOON

FURTHER MATHEMATICS – A2 unit 4
FURTHER PURE MATHEMATICS B

2 hours 30 minutes

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ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer **all** questions.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit.
Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.

INFORMATION FOR CANDIDATES

The maximum mark for this paper is 120.
The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

Reminder: Sufficient working must be shown to demonstrate the **mathematical** method employed.

1. A function f has domain $(-\infty, \infty)$ and is defined by $f(x) = \cosh^3 x - 3\cosh x$.

(a) Show that the graph of $y = f(x)$ has only one stationary point. [5]

(b) Find the nature of this stationary point. [2]

(c) State the largest possible range of $f(x)$. [1]

2. When plotted on an Argand diagram, the four fourth roots of the complex number $9 - 3\sqrt{3}\,\mathrm{i}$ lie on a circle. Find the equation of this circle. [4]

3. (a) By putting $t = \tan\left(\frac{\theta}{2}\right)$, show that the equation

$$4\sin\theta + 5\cos\theta = 3$$

can be written in the form

$$4t^2 - 4t - 1 = 0.$$ [3]

(b) Hence find the general solution of the equation

$$4\sin\theta + 5\cos\theta = 3.$$ [6]

4. The region R is bounded by the curve $x = \sin y$, the y-axis and the lines $y = 1$, $y = 3$. Find the volume of the solid generated when R is rotated through four right angles about the y-axis. Give your answer correct to two decimal places. [5]

5. (a) Determine the number of solutions of the equations

$$\begin{aligned} x + 2y &= 3, \\ 2x - 5y + 3z &= 8, \\ 6y - 2z &= 0. \end{aligned}$$ [4]

(b) Give a geometric interpretation of your answer in part (a). [1]

6. Solve the equation

$$\cos 2\theta - \cos 4\theta = \sin 3\theta \quad \text{for} \quad 0 \leqslant \theta \leqslant \pi.$$ [6]

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7. (a) Express $4x^2 + 10x - 24$ in the form $a(x+b)^2 + c$, where a, b, c are constants whose values are to be found. [3]

(b) Hence evaluate the integral

$$\int_3^5 \frac{6}{\sqrt{4x^2 + 10x - 24}}\,\mathrm{d}x\,.$$

Give your answer correct to 3 decimal places. [5]

8. By writing $x = \sinh y$, show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$. [6]

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9. (a) (i) Expand $\left(\cos\dfrac{\theta}{3} + \mathrm{i}\sin\dfrac{\theta}{3}\right)^3$.

(ii) Hence, by using de Moivre's theorem, show that $\cos\theta$ can be expressed as $4\cos^3\dfrac{\theta}{3} - 3\cos\dfrac{\theta}{3}$. [6]

(b) Hence, or otherwise, find the general solution of the equation $\dfrac{\cos\theta}{\cos\dfrac{\theta}{3}} = 1$. [6]

10. The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & 8 & 0 \\ 0 & \lambda & -2 \\ 4 & 0 & \lambda \end{pmatrix}.$$

(a) Show that there are two values of λ for which $\mathbf{A}$ is singular. [4]

(b) Given that $\lambda = 3$,

(i) determine the adjugate matrix of $\mathbf{A}$,

(ii) determine the inverse matrix $\mathbf{A}^{-1}$. [5]

11. (a) Differentiate each of the following with respect to x.

(i) $y = \mathrm{e}^{3x}\sin^{-1}x$

(ii) $y = \ln\left(\cosh^2\left(2x^2 + 7x\right)\right)$ [7]

(b) Find the equations of the tangents to the curve $x = \sinh^{-1}\left(y^2\right)$ at the points where $x = 1$. [8]

12. Find the solution of the differential equation

$$3\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 5\frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 8 + 6x - 2x^2 ,$$

where $y = 6$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 5$ when $x = 0$. [12]

13. The curve *C* has polar equation $r = 2 - \cos\theta$ for $0 \leqslant \theta \leqslant 2\pi$.

(a) Sketch the curve *C*. [2]

(b) (i) Show that the values of θ at which the tangent to the curve $r = 2 - \cos\theta$ is parallel to the initial line satisfy the equation

$$2\cos^2\theta - 2\cos\theta - 1 = 0 .$$

(ii) Find the polar coordinates of the points where the tangent to the curve $r = 2 - \cos\theta$ is parallel to the initial line. [9]

14. Evaluate the integral

$$\int_2^4 \frac{6x^2 + 2x + 16}{x^3 - x^2 + 3x - 3}\,\mathrm{d}x ,$$

giving your answer correct to three decimal places. [10]

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